MATH 54 – MOCK MIDTERM 2

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Name: ____

Instructions: This is a mock midterm and it's designed to give you an idea of what the actual midterm will look like.

1		10
2		20
3		5
4		5
5		10
6		10
7		10
8		5
9		10
10		10
11		5
Total		100

Date: Friday, July 13th, 2012.

1. *(10 points, 2 points each)*

Label the following statements as **T** or **F**.

NOTE: In this question, you do **NOT** have to show your work! Don't spend *too* much time on each question!

(a) If $dim(V) = 3$ and $\mathbf{u}$ and $\mathbf{v}$ are two vectors in V, then $\{\mathbf{u}, \mathbf{v}\}$ cannot be linearly independent!

(b) If T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, and T is onto, then T is also one-to-one.

(c) If A is a $m \times n$ matrix, then $Col(A)$ is a subspace of $\mathbb{R}^n$.

(d) If $\mathcal{C} \overset{P}{\leftarrow} \mathcal{B}$ is the change-of-coordinates matrix from $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ to $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$ then $\mathcal{C} \overset{P}{\leftarrow} \mathcal{B} = \begin{bmatrix} [\mathbf{c_1}]_{\mathcal{B}} & [\mathbf{c_2}]_{\mathcal{B}} \end{bmatrix}$

(e) The Span of any set of vectors is always a vector space.

2. *(20 points, 5 points each)* Label the following statements as **TRUE** or **FALSE**. In this question, you **HAVE** to justify your answer!!!

This means:

- If the answer is **TRUE**, you have to explain **WHY** it is true (possibly by citing a theorem)

- If the answer is **FALSE**, you have to give a specific **COUNTEREXAMPLE**. You also have to explain why the counterexample is in fact a counterexample to the statement!

(a) The set of 2×2 matrices such that $det(A) = 0$ is a vector space.

(b) A 4×5 matrix A cannot be invertible

Hint: How big is $Nul(A)$?

(c) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, the set of 2×2 matrices B such that $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a vector space.

(d) The set $\{1 - 2t + t^2, 3 - 5t + 4t^2, 2t + 3t^2\}$ is a basis for P_2

3. *(5 points)* Find the matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ which first reflects points in $\mathbb{R}^2$ about the line $y = x$ and then rotates them by 180 degrees (π radians) counterclockwise.

4. *(5 points)* A 2×2 matrix is called **symmetric** if $A^T = A$. Find a basis for the vector space V of all 2×2 symmetric matrices. Show that the basis you found is in fact a basis!

Hint: What does a general 2×2 symmetric matrix look like?

5. *(10 points)* For the following matrix A, find a basis for $Nul(A)$, $Row(A)$, $Col(A)$, and find $Rank(A)$:

$$A = \begin{bmatrix} 3 & -1 & 7 & 3 & 9 \\ -2 & 2 & -2 & 7 & 6 \\ -5 & 9 & 3 & 3 & 4 \\ -2 & 6 & 6 & 3 & 7 \end{bmatrix} \sim \begin{bmatrix} 3 & -1 & 7 & 3 & 9 \\ 0 & 2 & 4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6. *(10 points)* Let $\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right\}$, and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ be bases for $\mathbb{R}^2$.

(a) Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, namely: $\mathcal{C} \overset{P}{\leftarrow} \mathcal{B}$

(b) Calculate $[\mathbf{x}]_{\mathcal{C}}$ given $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

7. *(10 points)* Let $V = Span\left\{e^x, e^x\cos(x), e^x\sin(x)\right\}$, and define $T : V \to V$ by:

$$T(y) = y' + y$$

(a) Show T is linear

(b) Find the matrix of T with respect to the basis $\mathcal{B} = \left\{e^x, e^x\cos(x), e^x\sin(x)\right\}$ for V.

Note: Don't freak out! I know this is a brand new problem, but just do the same think you usually do to find matrices of linear transformations!

8. *(5 points)* Find the largest interval (a, b) on which the following differential equation has a unique solution:

$$\sin(x)y'' + \left(\sqrt{2-x}\right)y' = e^x$$

with

$$y\left(\frac{\pi}{2}\right) = 4, y'\left(\frac{\pi}{2}\right) = 0$$

9. *(10 points)* Solve the following differential equation:

$$y''' - 12y'' + 41y' - 42y = 0$$

Hint: $42 = 2 \times 3 \times 7$

10. *(10 points)*

(a) Solve $y'' + 4y' + 4y = e^{3t}$ using undetermined coefficients

(b) Solve $y'' + y = \tan(t)$ using variation of parameters

Note: You may need to use the fact that $\tan(t) = \frac{\sin(t)}{\cos(t)}$. Also you may use the fact that $\int \frac{\sin^2(t)}{\cos(t)} dt = \ln\left|\frac{\cos(t)}{\sin(t)-1}\right| - \sin(t)$.

11. *(5 points)* Suppose $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly dependent vectors (in V) and $T : V \to W$ is a linear transformation. Show that $T(\mathbf{u}), T(\mathbf{v}), T(\mathbf{w})$ are also linearly dependent.

    Hint: Write down what it means for 3 vectors to be linearly dependent!